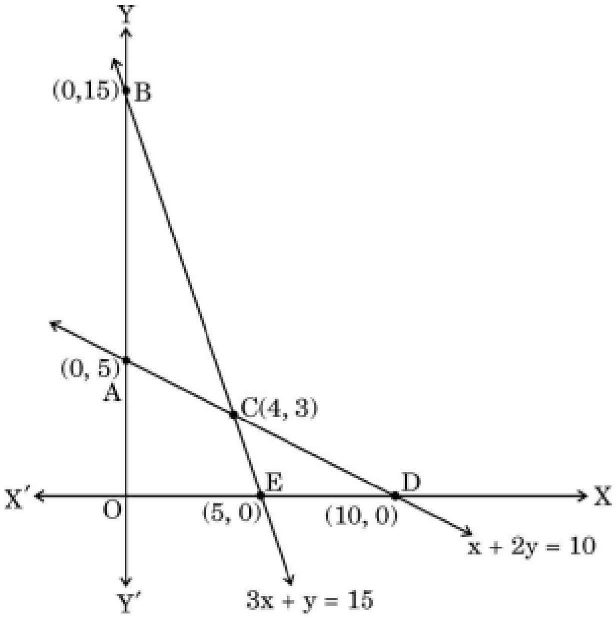
Marking Scheme **Strictly Confidential** **(For Internal and Restricted use only)** **Senior Secondary Examination, 2025** **SUBJECT: MATHEMATICS (Q.P. CODE – 65/5/1)**	
General Instructions: -	
1	You are aware that evaluation is the most important process in the actual and correct assessment of the candidates. A small mistake in evaluation may lead to serious problems which may affect the future of the candidates, education system and teaching profession. To avoid mistakes, it is requested that before starting evaluation, you must read and understand the spot evaluation guidelines carefully.
2	**"Evaluation policy is a confidential policy as it is related to the confidentiality of the examinations conducted, Evaluation done and several other aspects. Its leakage to the public in any manner could lead to derailment of the examination system and affect the life and future of millions of candidates. Sharing this policy/document to anyone, publishing in any magazine and printing in Newspaper/Website, etc. may invite action under various rules of the Board and IPC."**
3	Evaluation is to be done as per instructions provided in the Marking Scheme. It should not be done according to one's own interpretation or any other consideration. The Marking Scheme should be strictly adhered to and religiously followed. **However, while evaluating, answers which are based on latest information or knowledge and/or are innovative, they may be assessed for their correctness otherwise and due marks be awarded to them. In class-XII, while evaluating the competency-based questions, please try to understand the given answer and even if reply is not from a marking scheme but correct competency is enumerated by the candidate, due marks should be awarded.**
4	The Marking Scheme carries only suggested value points for the answers. These are Guidelines only and do not constitute the complete answer. The students can have their own expression and if the expression is correct, the due marks should be awarded accordingly.
5	The Head-Examiner must go through the first five answer books evaluated by each evaluator on the first day, to ensure that evaluation has been carried out as per the instructions given in the Marking Scheme. If there is any variation, the same should be zero after deliberation and discussion. The remaining answer books meant for evaluation shall be given only after ensuring that there is no significant variation in the marking of individual evaluators.
6	Evaluators will mark (√) wherever answer is correct. For wrong answer CROSS 'X' be marked. Evaluators will not put right (✓) while evaluating which gives the impression that the answer is correct, and no marks are awarded. **This is the most common mistake which evaluators are committing.**
7	If a question has parts, please award marks on the right-hand side for each part. Marks awarded for different parts of the question should then be totaled up and written in the left-hand margin and encircled. This may be followed strictly.
8	If a question does not have any parts, marks must be awarded in the left-hand margin and encircled. This may also be followed strictly.
9	If a student has attempted an extra question, answer to the question deserving more marks should be retained and the other answer scored out with a note **"Extra Question"**.

10	No marks to be deducted for the cumulative effect of an error. It should be penalized only once.
11	A full scale of marks_(example 0 to 80/70/60/50/40/30 marks as given in Question Paper) has to be used. Please do not hesitate to award full marks if the answer deserves it.
12	Every examiner must necessarily do evaluation work for full working hours, i.e., 8 hours every day and evaluate 20 answer books per day in main subjects and 25 answer books per day in other subjects (Details are given in Spot Guidelines). This is in view of the reduced syllabus and number of questions in question paper.
13	Ensure that you do not make the following common types of errors committed by the Examiner in the past: - ● Leaving answer or part thereof unassessed in an answer book. ● Giving more marks for an answer than assigned to it. ● Wrong totaling of marks awarded on an answer. ● Wrong transfer of marks from the inside pages of the answer book to the title page. ● Wrong question wise totaling on the title page. ● Wrong totaling of marks of the two columns on the title page. ● Wrong grand total. ● Marks in words and figures not tallying/not same. ● Wrong transfer of marks from the answer book to online award list. ● Answers marked as correct, but marks not awarded. (Ensure that the right tick mark is correctly and clearly indicated. It should merely be a line. Same is with the X for incorrect answer.) ● Half or a part of the answer marked correct and the rest as wrong, but no marks awarded.
14	While evaluating the answer books if the answer is found to be totally incorrect, it should be marked as cross (X) and awarded zero (0) Marks.
15	Any unassessed portion, non-carrying over of marks to the title page, or total error detected by the candidate shall damage the prestige of all the personnel engaged in the evaluation work as also of the Board. Hence, to uphold the prestige of all concerned, it is again reiterated that the instructions be followed meticulously and judiciously.
16	The Examiners should acquaint themselves with the guidelines given in the "**Guidelines for Spot Evaluation**" before starting the actual evaluation.
17	Every Examiner shall also ensure that all the answers are evaluated, marks carried over to the title page, correctly totaled and written in figures and words.
18	The candidates are entitled to obtain a photocopy of the Answer Book on request on payment of the prescribed processing fee. All Examiners/Additional Head Examiners/Head Examiners are once again reminded that they must ensure that evaluation is carried out strictly as per value points for each answer as given in the Marking Scheme.

Q.No.	EXPECTED ANSWER / VALUE POINTS	Marks
	SECTION-A ***This section comprises multiple choice questions (MCQs) of 1 mark each.***	
1.	If $A = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$, then A^3 is : (A) $3\begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$ (B) $\begin{bmatrix} 125 & 0 & 0 \\ 0 & 125 & 0 \\ 0 & 0 & 125 \end{bmatrix}$ (C) $\begin{bmatrix} 15 & 0 & 0 \\ 0 & 15 & 0 \\ 0 & 0 & 15 \end{bmatrix}$ (D) $\begin{bmatrix} 5^3 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$	
Ans	(B) $\begin{bmatrix} 125 & 0 & 0 \\ 0 & 125 & 0 \\ 0 & 0 & 125 \end{bmatrix}$	**1**
2.	If $P(A \cup B) = 0{\cdot}9$ and $P(A \cap B) = 0{\cdot}4$, then $P(\overline{A}) + P(\overline{B})$ is : (A) 0·3 (B) 1 (C) 1·3 (D) 0·7	
Ans	**(D) 0.7**	**1**
3.	If $A = \begin{bmatrix} 1 & 2 & 3 \\ -4 & 3 & 7 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 3 \\ -1 & 2 \\ 0 & 5 \end{bmatrix}$, then the correct statement is : (A) Only AB is defined. (B) Only BA is defined. (C) AB and BA, both are defined. (D) AB and BA, both are not defined.	
Ans	**(C) AB and BA, both are defined.**	**1**
4.	If $\begin{vmatrix} 2x & 5 \\ 12 & x \end{vmatrix} = \begin{vmatrix} 6 & -5 \\ 4 & 3 \end{vmatrix}$, then the value of x is : (A) 3 (B) 7 (C) ± 7 (D) ± 3	
Ans	**(C) ± 7**	**1**

5.	If $f(x) = \begin{cases} \frac{\sin^2 ax}{x^2}, & x \neq 0 \\ 1, & x = 0 \end{cases}$ is continuous at x = 0, then the value of a is : (A) 1 (B) –1 (C) ± 1 (D) 0	
Ans	**(C) ±1**	**1**
6.	If A = $[a_{ij}]$ is a 3 × 3 diagonal matrix such that $a_{11} = 1$, $a_{22} = 5$ and $a_{33} = -2$, then $\lvert A \rvert$ is : (A) 0 (B) –10 (C) 10 (D) 1	
Ans	**(B) -10**	**1**
7.	The principal value of $\cot^{-1}\left(-\frac{1}{\sqrt{3}}\right)$ is : (A) $-\frac{\pi}{3}$ (B) $-\frac{2\pi}{3}$ (C) $\frac{\pi}{3}$ (D) $\frac{2\pi}{3}$	
Ans	**(D)** $\frac{2\pi}{3}$	**1**
8.	If $\begin{bmatrix} 4+x & x-1 \\ -2 & 3 \end{bmatrix}$ is a singular matrix, then the value of x is : (A) 0 (B) 1 (C) – 2 (D) – 4	
Ans	**(C) -2**	**1**
9.	If f(x) = {[x], x ∈ R} is the greatest integer function, then the correct statement is : (A) f is continuous but not differentiable at x = 2. (B) f is neither continuous nor differentiable at x = 2. (C) f is continuous as well as differentiable at x = 2. (D) f is not continuous but differentiable at x = 2.	
Ans	**(B) f is neither continuous nor differentiable at x=2.**	**1**

10.	The slope of the curve $y = -x^3 + 3x^2 + 8x - 20$ is maximum at : (A) (1, –10) (B) (1, 10) (C) (10, 1) (D) (–10, 1)	
Ans	**(A) (1, -10)**	**1**
11.	$\int \sqrt{1+\sin x}\ dx$ is equal to : (A) $2\left(-\sin\frac{x}{2}+\cos\frac{x}{2}\right)+C$ (B) $2\left(\sin\frac{x}{2}-\cos\frac{x}{2}\right)+C$ (C) $-2\left(\sin\frac{x}{2}+\cos\frac{x}{2}\right)+C$ (D) $2\left(\sin\frac{x}{2}+\cos\frac{x}{2}\right)+C$	
Ans	**(B) $2\left(\sin\frac{x}{2}-\cos\frac{x}{2}\right)+C$**	1
12.	$\int_0^{\pi/2} \cos x . e^{\sin x}\ dx$ is equal to : (A) 0 (B) $1-e$ (C) $e-1$ (D) e	
Ans	**(C) e - 1**	**1**
13.	The area of the region enclosed between the curve $y = x\|x\|$, x-axis, $x = -2$ and $x = 2$ is : (A) $\frac{8}{3}$ (B) $\frac{16}{3}$ (C) 0 (D) 8	
Ans	**(B) $\frac{16}{3}$**	**1**
14.	The integrating factor of the differential equation $\left(\frac{e^{-2\sqrt{x}}}{\sqrt{x}}-\frac{y}{\sqrt{x}}\right)\frac{dx}{dy}=1$ is : (A) $e^{-1/\sqrt{x}}$ (B) $e^{2/\sqrt{x}}$ (C) $e^{2\sqrt{x}}$ (D) $e^{-2\sqrt{x}}$	
Ans	**(C) $e^{2\sqrt{x}}$**	**1**

15.	The sum of the order and degree of the differential equation $\left[1+\left(\frac{dy}{dx}\right)^2\right]^3 = \frac{d^2y}{dx^2}$ is : (A) 2 (B) $\frac{5}{2}$ (C) 3 (D) 4	
Ans	**(C) 3**	**1**
16.	For a Linear Programming Problem (LPP), the given objective function Z = 3x + 2y is subject to constraints : $x + 2y \le 10$ $3x + y \le 15$ $x, y \ge 0$  The correct feasible region is : (A) ABC (B) AOEC (C) CED (D) Open unbounded region BCD	
Ans	**(B) AOEC**	**1**
17.	Let $\vec{a}$ be a position vector whose tip is the point (2, – 3). If $\overrightarrow{AB} = \vec{a}$, where coordinates of A are (– 4, 5), then the coordinates of B are : (A) (– 2, – 2) (B) (2, – 2) (C) (– 2, 2) (D) (2, 2)	
Ans	**(C) (−2, 2)**	**1**
18.	The respective values of $\lvert\vec{a}\rvert$ and $\lvert\vec{b}\rvert$, if given $(\vec{a} - \vec{b}) \cdot (\vec{a} + \vec{b}) = 512$ and $\lvert\vec{a}\rvert = 3\lvert\vec{b}\rvert$, are : (A) 48 and 16 (B) 3 and 1 (C) 24 and 8 (D) 6 and 2	
Ans	**(C) 24 and 8**	**1**

*Questions number **19** and **20** are Assertion and Reason based questions. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (A), (B), (C) and (D) as given below.*

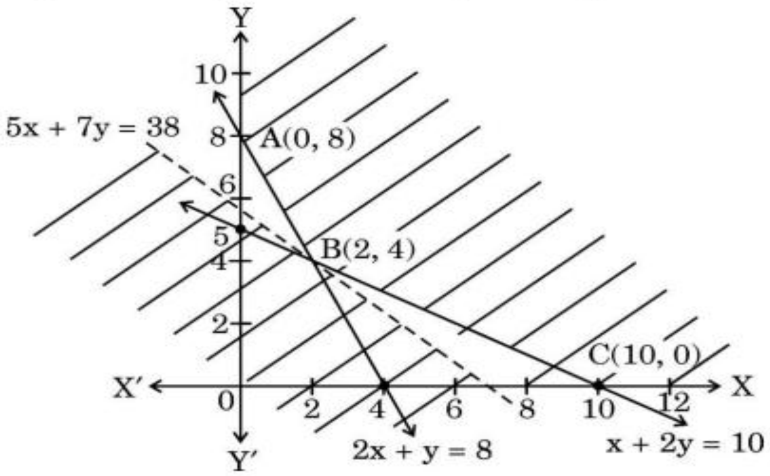
(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is ***not*** the correct explanation of the Assertion (A).

(C) Assertion (A) is true, but Reason (R) is false.

(D) Assertion (A) is false, but Reason (R) is true.

19. *Assertion (A) :* The shaded portion of the graph represents the feasible region for the given Linear Programming Problem (LPP).



Min $Z = 50x + 70y$

subject to constraints

$2x + y \geq 8$, $x + 2y \geq 10$, $x, y \geq 0$

$Z = 50x + 70y$ has a minimum value $= 380$ at $B(2, 4)$.

Reason (R) : The region representing $50x + 70y < 380$ does not have any point common with the feasible region.

Ans	**(A) Both Assertion (A) and Reason (R) are true, and Reason (R) is the correct explanation of the Assertion (A).**	**1**

20. *Assertion (A) :* Let $A = \{x \in R : -1 \leq x \leq 1\}$. If $f : A \to A$ be defined as $f(x) = x^2$, then f is not an onto function.

Reason (R) : If $y = -1 \in A$, then $x = \pm\sqrt{-1} \notin A$.

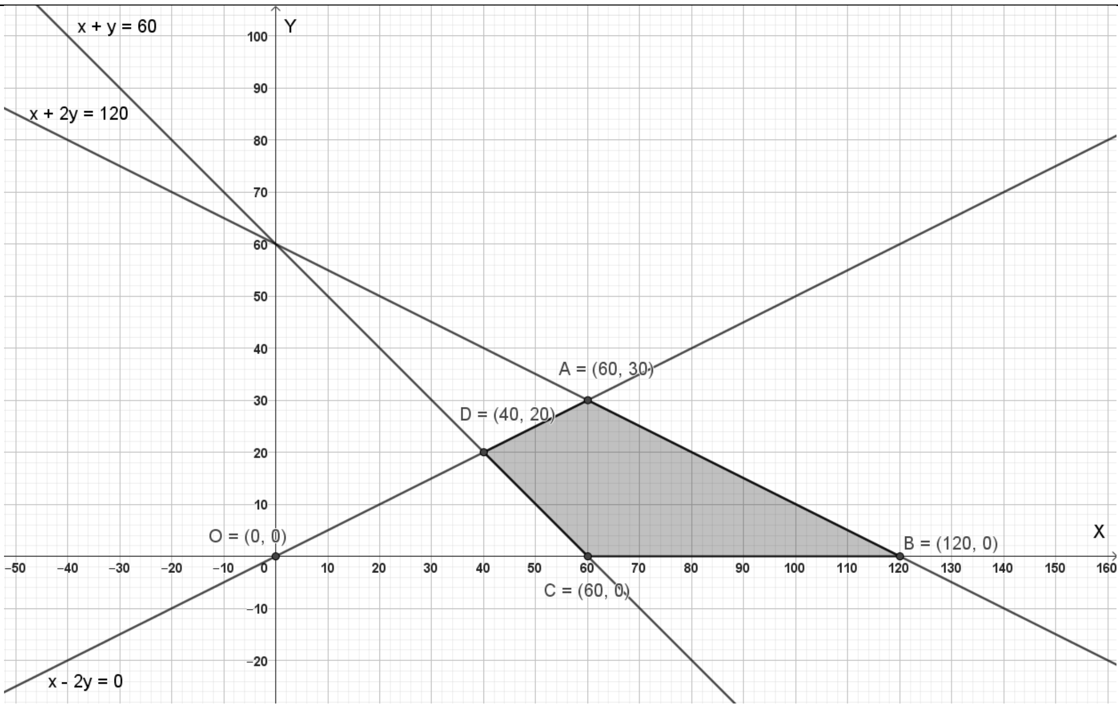
Ans	**(A) Both Assertion (A) and Reason (R) are true, and Reason (R) is the correct explanation of the Assertion (A).**	**1**

SECTION-B *This section comprises 5 Very Short Answer (VSA) type questions of 2 marks each.*		
21.	Find the domain of the function $f(x) = \cos^{-1}(x^2 - 4)$.	
Ans	**Domain of** $\cos^{-1}x$ **is** $[-1, 1]$ $\Rightarrow -1 \leq x^2 - 4 \leq 1 \Rightarrow 3 \leq x^2 \leq 5$ $\Rightarrow x \in [-\sqrt{5}, -\sqrt{3}] \cup [\sqrt{3}, \sqrt{5}]$	**1** ½ ½
22.	Surface area of a balloon (spherical), when air is blown into it, increases at a rate of 5 mm^2/s. When the radius of the balloon is 8 mm, find the rate at which the volume of the balloon is increasing.	
Ans	$\frac{dS}{dt} = 5\ mm^2/s$, $\left(\frac{dV}{dt}\right)_{r=8}$ =? $S=4\pi r^2 \Rightarrow \frac{dS}{dt} = 8\pi r.\frac{dr}{dt} \Rightarrow \frac{dr}{dt} = \frac{5}{8\pi r}$ $V=\frac{4}{3}\pi r^3 \Rightarrow \frac{dV}{dt} = 4\pi r^2.\frac{dr}{dt} \Rightarrow \frac{dV}{dt} = \frac{5}{2}r$ $\Rightarrow \left(\frac{dV}{dt}\right)_{r=8} = 20\ mm^3/s$	 ½ **1** ½
23.	(a) Differentiate $\frac{\sin x}{\sqrt{\cos x}}$ with respect to x. **OR** (b) If $y = 5\cos x - 3\sin x$, prove that $\frac{d^2y}{dx^2} + y = 0$.	
Ans	**(a) Let** $y = \frac{\sin x}{\sqrt{\cos x}}$ $\frac{dy}{dx} = \frac{\sqrt{\cos x}.\cos x - \sin x.\left(\frac{-\sin x}{2\sqrt{\cos x}}\right)}{\cos x}$ $\Rightarrow \frac{dy}{dx} = \frac{2\cos^2x + \sin^2x}{2(\cos x)^{3/2}}$ or $\frac{1+\cos^2x}{2(\cos x)^{3/2}}$ **OR** **(b)** $y = 5\cos x - 3\sin x$, **then** $\frac{dy}{dx} = -5.\sin x - 3.\cos x$ $\Rightarrow \frac{d^2y}{dx^2} = -5.\cos x + 3.\sin x = -y$ $\Rightarrow \frac{d^2y}{dx^2} + y = 0$	 **1½** ½ **1** ½ ½

24.	(a) Find a vector of magnitude 5 which is perpendicular to both the vectors $3\hat{i} - 2\hat{j} + \hat{k}$ and $4\hat{i} + 3\hat{j} - 2\hat{k}$. **OR** (b) Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three vectors such that $\vec{a}.\vec{b} = \vec{a}.\vec{c}$ and $\vec{a} \times \vec{b} = \vec{a} \times \vec{c}$, $\vec{a} \neq 0$. Show that $\vec{b} = \vec{c}$.	
Ans	**Let** $\vec{a} = 3\hat{i} - 2\hat{j} + \hat{k}$, $\vec{b} = 4\hat{i} + 3\hat{j} - 2\hat{k}$	
	$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -2 & 1 \\ 4 & 3 & -2 \end{vmatrix} = \hat{i} + 10\hat{j} + 17\hat{k}$	½
	$\lvert\vec{a} \times \vec{b}\rvert = \sqrt{1^2 + 10^2 + 17^2} = \sqrt{390}$	½
	Unit vector $\hat{n} = \frac{\vec{a}\times\vec{b}}{\lvert\vec{a}\times\vec{b}\rvert} = \frac{1}{\sqrt{390}}(\hat{i} + 10\hat{j} + 17\hat{k})$	½
	∴ **Required vector** $= \frac{5}{\sqrt{390}}(\hat{i} + 10\hat{j} + 17\hat{k})$	½
	OR	
	(b) $\vec{a}.\vec{b} = \vec{a}.\vec{c} \Rightarrow \vec{a}.(\vec{b} - \vec{c}) = 0$	
	$\Rightarrow$ **either** $\vec{b} = \vec{c}$ **or** $\vec{a} \perp (\vec{b} - \vec{c})$, **since** $\vec{a} \neq 0$	1
	Also, $\vec{a} \times \vec{b} = \vec{a} \times \vec{c} \Rightarrow \vec{a} \times (\vec{b} - \vec{c}) = 0$	
	$\Rightarrow$ **either** $\vec{b} = \vec{c}$ **or** $\vec{a} \parallel (\vec{b} - \vec{c})$, **since** $\vec{a} \neq 0$	½
	Since vectors $\vec{a}$ **and** $(\vec{b} - \vec{c})$ **cannot be** $\parallel$ **and** $\perp$ **simultaneously**	
	Hence $\vec{b} = \vec{c}$	½
25.	A man needs to hang two lanterns on a straight wire whose end points have coordinates A (4, 1, – 2) and B (6, 2, – 3). Find the coordinates of the points where he hangs the lanterns such that these points trisect the wire AB.	
Ans		
	Let P and Q trisect the wire AB.	
	P divides AB in the ratio 1:2 then, coordinate of point P $= \left(\frac{14}{3}, \frac{4}{3}, -\frac{7}{3}\right)$	1
	Q divides AB in the ratio 2:1 then, coordinate of point Q $= \left(\frac{16}{3}, \frac{5}{3}, -\frac{8}{3}\right)$	1

SECTION-C *This section comprises 6 Short Answer (SA) type questions of 3 marks each.*		
26.	Find the value of 'a' for which f(x) = $\sqrt{3}\sin x - \cos x - 2ax + 6$ is decreasing in R.	
Ans	**Since** $f(x)$ **is a decreasing function** $\Rightarrow f'(x) \le 0$ $\Rightarrow \sqrt{3}.\cos x + \sin x - 2a \le 0$ $\Rightarrow 2\left(\frac{\sqrt{3}}{2}.\cos x + \frac{1}{2}\sin x\right) - 2a \le 0$ $\Rightarrow \cos\left(x - \frac{\pi}{6}\right) \le a$ **Since,** $-1 \le \cos\left(x - \frac{\pi}{6}\right) \le 1 \Rightarrow a \ge 1$ i.e. $a \epsilon [1, \infty)$ or $(1, \infty)$	 1 1 1
27.	(a) Find : $$\int \frac{2x}{(x^2+3)(x^2-5)}dx$$ **OR** (b) Evaluate : $$\int_1^4 (\lvert x-2\rvert + \lvert x-4\rvert)\ dx$$	
Ans	**(a)** Let $I = \int \frac{2x}{(x^2+3)(x^2-5)}dx$ **Put** $x^2 = t \Rightarrow 2x.dx = dt$ $\Longrightarrow I = \int \frac{dt}{(t+3)(t-5)}$ $= \int\left(-\frac{1}{8(t+3)} + \frac{1}{8(t-5)}\right)dt$ $= \frac{1}{8}[\log\lvert t-5\rvert - \log\lvert t+3\rvert] + c$ $= \frac{1}{8}\log\left\lvert\frac{x^2-5}{x^2+3}\right\rvert + c$ **OR** **(b)** $\int_1^4(\lvert x-2\rvert + \lvert x-4\rvert)\,dx$ $= \int_1^2(2-x)\,dx + \int_2^4(x-2)dx - \int_1^4(x-4)\,dx$ $= \left[\frac{(2-x)^2}{-2}\right]_1^2 + \left[\frac{(x-2)^2}{2}\right]_2^4 - \left[\frac{(x-4)^2}{2}\right]_1^4$ $= \frac{1}{2} + 2 + \frac{9}{2} = 7$	 ½ 1 1 ½ 1½ 1 ½

28.	Find the particular solution of the differential equation $$\left[x \sin^2\left(\frac{y}{x}\right) - y\right] dx + x\, dy = 0$$ given that $y = \frac{\pi}{4}$, when $x = 1$.	
Ans	$\left[x.\sin^2\frac{y}{x} - y\right].dx + x.dy = 0$ $\Rightarrow \frac{dy}{dx} = \frac{y}{x} - \sin^2\frac{y}{x}$ Put $y = vx \implies \frac{dy}{dx} = x\frac{dv}{dx} + v$ $\therefore\ x\frac{dv}{dx} + v = v - \sin^2 v$ $\implies -\int \text{cosec}^2 v\, dv = \int \frac{dx}{x}$ $\implies \cot v = \log\lvert x\rvert + c$ $\implies \cot\frac{y}{x} = \log\lvert x\rvert + c$ $x = 1, y = \frac{\pi}{4} \implies c = 1$ $\implies \cot\frac{y}{x} = \log\lvert x\rvert + 1$	1 1 ½ ½
29.	In the Linear Programming Problem (LPP), find the point/points giving maximum value for Z = 5x + 10y subject to constraints $x + 2y \le 120$ $x + y \ge 60$ $x - 2y \ge 0$ $x, y \ge 0$	
Ans		



1½ for correct graph and correct feasible region

Corner Points	Value of Z
A (60, 30)	600
B (120, 0)	600
C (60, 0)	300
D (40, 20)	400

1

Since Z is maximum on points A and B

Hence all points lying on segment AB give maximum Z. **½**

30. (a) If $\vec{a} + \vec{b} + \vec{c} = \vec{0}$ such that $|\vec{a}| = 3$, $|\vec{b}| = 5$, $|\vec{c}| = 7$, then find the angle between $\vec{a}$ and $\vec{b}$.

OR

(b) If $\vec{a}$ and $\vec{b}$ are unit vectors inclined with each other at an angle θ, then prove that $\frac{1}{2}|\vec{a} - \vec{b}| = \sin\frac{\theta}{2}$.

Ans

Given $\vec{a} + \vec{b} + \vec{c} = \vec{0} \Rightarrow |\vec{a} + \vec{b}| = |-\vec{c}|$ **1**

$\Rightarrow |\vec{a} + \vec{b}|^2 = |\vec{c}|^2 \Rightarrow |\vec{a}|^2 + |\vec{b}|^2 + 2\,\vec{a}.\vec{b} = |\vec{c}|^2$

$\Rightarrow 9 + 25 + 2\,\vec{a}.\vec{b} = 49$ **1**

	$\Rightarrow 2\|\vec{a}\|\|\vec{b}\|\cos\theta = 15$	
	$\Rightarrow \cos\theta = \frac{1}{2} \therefore\ \theta = \frac{\pi}{3}$	1
	OR	
	(b) $\|\vec{a}\| = \|\vec{b}\| = 1$	½
	$\|\vec{a}-\vec{b}\|^2 = \|\vec{a}\|^2 + \|\vec{b}\|^2 - 2\vec{a}.\vec{b}$	1
	$= 1 + 1 - 2\|\vec{a}\|\|\vec{b}\|\cos\theta$	½
	$= 2 - 2\cos\theta$	
	$= 2\left(2\sin^2\frac{\theta}{2}\right) = 4\sin^2\frac{\theta}{2}$	½
	$\Rightarrow \sin\frac{\theta}{2} = \frac{1}{2}\|\vec{a}-\vec{b}\|$	½
31.	(a) The probability that a student buys a colouring book is 0·7 and that she buys a box of colours is 0·2. The probability that she buys a colouring book, given that she buys a box of colours, is 0·3. Find the probability that the student : (i) Buys both the colouring book and the box of colours. (ii) Buys a box of colours given that she buys the colouring book. **OR** (b) A person has a fruit box that contains 6 apples and 4 oranges. He picks out a fruit three times, one after the other, after replacing the previous one in the box. Find : (i) The probability distribution of the number of oranges he draws. (ii) The expectation of the random variable (number of oranges).	
Ans	**(a) Let A be the event of buying colouring book and B be the event of buying coloured box.**	½
	$P(A) = 0.7,\quad P(B) = 0.2,\quad P(A/B) = 0.3$	½
	(i) $P\left(\frac{A}{B}\right) = \frac{P(A\cap B)}{P(B)} \Rightarrow 0.3 = \frac{P(A\cap B)}{0.2}$	
	$\Rightarrow P(A\cap B) = 0.06$ **or** $\frac{3}{50}$	1

(ii) $P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)}$

$= \frac{0.06}{0.7} = \frac{3}{35}$ or 0.086 — **1**

OR

(b) Let X be random variable for number of oranges.

X = 0, 1, 2, 3 — ½

Let A be the event that orange is drawn.

$P(A) = \frac{4}{10} = \frac{2}{5}$, $P(\bar{A}) = 1 - \frac{2}{5} = \frac{3}{5}$ — ½

(i)

X	0	1	2	3
P(X)	$\frac{27}{125}$	$\frac{54}{125}$	$\frac{36}{125}$	$\frac{8}{125}$

1

(ii) $E(X) = \sum p_i x_i = 0 \times \frac{27}{125} + 1 \times \frac{54}{125} + 2 \times \frac{36}{125} + 3 \times \frac{8}{125}$ — ½

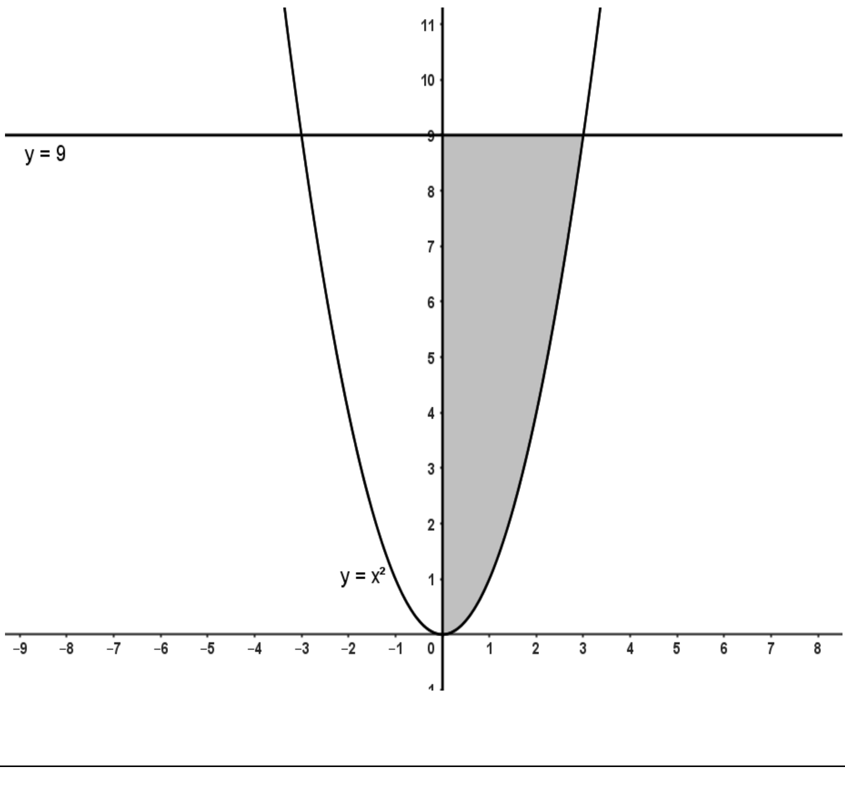
$= \frac{150}{125}$ or $\frac{6}{5}$ — ½

SECTION-D

This section comprises 4 Long Answer (LA) type questions of 5 marks each.

32. Sketch a graph of $y = x^2$. Using integration, find the area of the region bounded by $y = 9$, $x = 0$ and $y = x^2$.

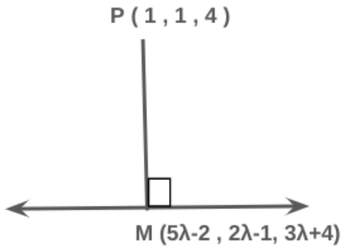
Ans



1½ for correct figure and shading

	Required area $= \int_0^9 \sqrt{y}\ dy$ $= \frac{2}{3}\left[y^{3/2}\right]_0^9$ $= 18$ **Note: If area is found in second quadrant, may be considered.**	1½ 1 1
33.	A furniture workshop produces three types of furniture – chairs, tables and beds each day. On a particular day the total number of furniture pieces produced is 45. It was also found that production of beds exceeds that of chairs by 8, while the total production of beds and chairs together is twice the production of tables. Determine the units produced of each type of furniture, using matrix method.	
Ans	**Let the numbers of chairs, tables and beds produced be x, y and z respectively.** $\therefore\ x + y + z = 45;\quad -x + 0.y + z = 8;\quad x - 2y + z = 0$ **Let** $A = \begin{bmatrix} 1 & 1 & 1 \\ -1 & 0 & 1 \\ 1 & -2 & 1 \end{bmatrix},\ X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 45 \\ 8 \\ 0 \end{bmatrix}$ $\lvert A \rvert = 1(0+2) - 1(-1-1) + 1(2-0) = 6 \neq 0$ $\therefore A^{-1}$ **exists** $AX = B \Rightarrow X = A^{-1}B$ $adj(A) = \begin{bmatrix} 2 & -3 & 1 \\ 2 & 0 & -2 \\ 2 & 3 & 1 \end{bmatrix}$ $A^{-1} = \frac{1}{6}\begin{bmatrix} 2 & -3 & 1 \\ 2 & 0 & -2 \\ 2 & 3 & 1 \end{bmatrix}$ $\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{6}\begin{bmatrix} 2 & -3 & 1 \\ 2 & 0 & -2 \\ 2 & 3 & 1 \end{bmatrix}\begin{bmatrix} 45 \\ 8 \\ 0 \end{bmatrix} = \begin{bmatrix} 11 \\ 15 \\ 19 \end{bmatrix}$ **So,** $x = 11, y = 15, z = 19$ **Hence the numbers of chairs, tables and beds produced are 11, 15 and 19 respectively.**	1 ½ ½ ½ 1½ 1
34.	(a) For a positive constant 'a', differentiate $a^{t+\frac{1}{t}}$ with respect to $\left(t+\frac{1}{t}\right)^a$, where t is a non-zero real number. **OR** (b) Find $\frac{dy}{dx}$ if $y^x + x^y + x^x = a^b$, where a and b are constants.	

Ans	(a) Let $u = a^{t+\frac{1}{t}} \Rightarrow \frac{du}{dt} = a^{t+\frac{1}{t}}.\log a.\left(1-\frac{1}{t^2}\right)$ $v = \left(t+\frac{1}{t}\right)^a \Rightarrow \frac{dv}{dt} = a\left(t+\frac{1}{t}\right)^{a-1}.\left(1-\frac{1}{t^2}\right)$ $\frac{du}{dv} = \frac{du/dt}{dv/dt} = \frac{a^{t+\frac{1}{t}}.\log a}{a\left(t+\frac{1}{t}\right)^{a-1}}$ **OR** (b) Let $u = y^x$, $v = x^y$ and $w = x^x$ $\Rightarrow \frac{du}{dx} + \frac{dv}{dx} + \frac{dw}{dx} = 0$ ....(i) $u = y^x \Rightarrow \log u = x.\log y \Rightarrow \frac{1}{u}.\frac{du}{dx} = \frac{x}{y}.\frac{dy}{dx} + \log y$ $\Rightarrow \frac{du}{dx} = y^x\left(\frac{x}{y}.\frac{dy}{dx} + \log y\right) = xy^{x-1}\frac{dy}{dx} + y^x \log y$ $v = x^y \Rightarrow \log v = y.\log x \Rightarrow \frac{1}{v}.\frac{dv}{dx} = \frac{y}{x} + \log x.\frac{dy}{dx}$ $\Rightarrow \frac{dv}{dx} = x^y\left(\frac{y}{x} + \log x.\frac{dy}{dx}\right) = yx^{y-1} + x^y \log x\, \frac{dy}{dx}$ $w = x^x \Rightarrow \log w = x.\log x \Rightarrow \frac{1}{w}.\frac{dw}{dx} = 1 + \log x$ $\Rightarrow \frac{dw}{dx} = x^x.(1+\log x)$ ∴ From (i), we get $xy^{x-1}.\frac{dy}{dx} + y^x.\log y + yx^{y-1} + x^y.\log x.\frac{dy}{dx} + x^x.(1+\log x) = 0$ $\Rightarrow \frac{dy}{dx} = -\frac{x^x.(1+\log x) + y^x.\log y + yx^{y-1}}{x.y^{x-1} + x^y.\log x}$	2 2 1 1 1 1 1 1
35.	(a) Find the foot of the perpendicular drawn from the point (1, 1, 4) on the line $\frac{x+2}{5} = \frac{y+1}{2} = \frac{-z+4}{-3}$. **OR** (b) Find the point on the line $\frac{x-1}{3} = \frac{y+1}{2} = \frac{z-4}{3}$ at a distance of $2\sqrt{2}$ units from the point (–1, –1, 2).	
Ans	(a) Let $\frac{x+2}{5} = \frac{y+1}{2} = \frac{z-4}{3} = \lambda$ Coordinate of general point on the given line are $M\,(5\lambda-2, 2\lambda-1, 3\lambda+4)$	 1



Direction Ratios of PM vector are $< 5\lambda - 3, 2\lambda - 2, 3\lambda >$ **1**

Since, PM $\perp l$

$\Rightarrow 5(5\lambda - 3) + 2(2\lambda - 2) + 3(3\lambda) = 0$ **1**

$\Rightarrow \lambda = \frac{1}{2}$ **1**

Hence, coordinates of M are $\left(\frac{1}{2}, 0, \frac{11}{2}\right)$ **1**

OR

(b) Equation of given line be $\frac{x-1}{3} = \frac{y+1}{2} = \frac{z-4}{3} = \lambda$ **(say)**

Coordinate of any general point on the line are P $(3\lambda + 1, 2\lambda - 1, 3\lambda + 4)$**.** **1**

Let distance of point P from $(-1, -1, 2)$ **is** $2\sqrt{2}$**.**

$\Longrightarrow \sqrt{(3\lambda + 2)^2 + (2\lambda)^2 + (3\lambda + 2)^2} = 2\sqrt{2}$

$\Longrightarrow 22\lambda^2 + 24\lambda = 0$ **1½**

$\Rightarrow \lambda = 0$ **or** $\lambda = -\frac{12}{11}$ **1**

Hence, coordinates of point P are $(1, -1, 4)$ **or** $\left(-\frac{25}{11}, -\frac{35}{11}, \frac{8}{11}\right)$ **1½**

SECTION-E

This section comprises 3 case study-based questions of 4 marks each

36.

Case Study – 1

A carpenter needs to make a wooden cuboidal box, closed from all sides, which has a square base and fixed volume. Since he is short of the paint required to paint the box on completion, he wants the surface area to be minimum.

On the basis of the above information, answer the following questions :

(i) Taking length = breadth = x m and height = y m, express the surface area (S) of the box in terms of x and its volume (V), which is constant.

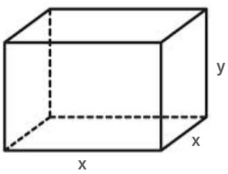
(ii) Find $\frac{dS}{dx}$.

(iii) (a) Find a relation between x and y such that the surface area (S) is minimum.

OR

(iii) (b) If surface area (S) is constant, the volume (V) = $\frac{1}{4}(Sx - 2x^3)$, x being the edge of base. Show that volume (V) is maximum for x = $\sqrt{\frac{S}{6}}$.

Ans



(i) $V = x^2y \Rightarrow y = \frac{V}{x^2} \ldots\ldots\ldots\ldots(i)$	
Hence, $S = 2x^2 + 4xy = 2x^2 + \frac{4V}{x}$	**1**
(ii) $\frac{dS}{dx} = 4\left(x - \frac{V}{x^2}\right)$	**1**
(iii) **(a)** $\frac{dS}{dx} = 0 \Rightarrow V = x^3 \Rightarrow x^2y = x^3 \Rightarrow y = x$	**1**
$\frac{d^2S}{dx^2} = 4\left(1 + \frac{2V}{x^3}\right) > 0 \Rightarrow$ **S is minimum if y = x.**	**1**
OR	
(iii) **(b)** $V = \frac{1}{4}(Sx - 2x^3) \Rightarrow \frac{dV}{dx} = \frac{1}{4}(S - 6x^2)$	**1**

	Put $\frac{dV}{dx}=0 \Rightarrow x=\sqrt{\frac{S}{6}}$ $\left(\frac{d^2V}{dx^2}\right)_{x=\sqrt{\frac{S}{6}}}=-3\sqrt{\frac{S}{6}}<0 \Rightarrow$ Volume is maximum for $x=\sqrt{\frac{S}{6}}$.	½ ½
37.	**Case Study – 2** Let A be the set of 30 students of class XII in a school. Let f : A → N, N is a set of natural numbers such that function f(x) = Roll Number of student x. On the basis of the given information, answer the following : (i) Is f a bijective function ? (ii) Give reasons to support your answer to (i). (iii) (a) Let R be a relation defined by the teacher to plan the seating arrangement of students in pairs, where R = {(x, y) : x, y are Roll Numbers of students such that y = 3x}. List the elements of R. Is the relation R reflexive, symmetric and transitive ? Justify your answer. **OR** (iii) (b) Let R be a relation defined by R = {(x, y) : x, y are Roll Numbers of students such that $y = x^3$}. List the elements of R. Is R a function ? Justify your answer.	
Ans	**(i)** No, f is not bijective function **(ii)** Range $=\{1,2,3,4,\ldots\ldots\ldots\ldots,30\}$ and codomain= N Since, Range ≠ codomain ⇒ f is not onto and hence f is not bijective. **(iii)** (a) $R=\{(1,3),(2,6),(3,9),(4,12),(5,15),(6,18),(7,21),(8,24),(9,27),(10,30)\}$ Since $(1,1)\notin R \Longrightarrow$ R is not reflexive. $(1,3)\in R$ but $(3,1)\notin R \Longrightarrow$ R is not symmetric $(1,3)\in R,(3,9)\in R$ but $(1,9)\notin R \Longrightarrow$ R is not transitive. **OR** **(iii)** (b) $R=\{(1,1),(2,8),(3,27)\}$ ∵ elements 4, 5, 6 … 30 do not have an image. Hence the above relation is not a function.	1 ½ ½ 1 1 1 1

38.

Case Study – 3

A gardener wanted to plant vegetables in his garden. Hence he bought 10 seeds of brinjal plant, 12 seeds of cabbage plant and 8 seeds of radish plant. The shopkeeper assured him of germination probabilities of brinjal, cabbage and radish to be 25%, 35% and 40% respectively. But before he could plant the seeds, they got mixed up in the bag and he had to sow them randomly.

Radish Cabbage Brinjal

Based upon the above information, answer the following questions :

(i) Calculate the probability of a randomly chosen seed to germinate.

(ii) What is the probability that it is a cabbage seed, given that the chosen seed germinates ?

Ans

Let A: Event that chosen seed germinates.

B: Event that Brinjal seed is chosen.

C: Event that Cabbage seed is chosen.

R: Event that Radish seed is chosen.

$$P(B) = \frac{10}{30};\ P(C) = \frac{12}{30};\ P(R) = \frac{8}{30};$$

$$P\left(\frac{A}{B}\right) = \frac{25}{100};\ P\left(\frac{A}{C}\right) = \frac{35}{100};\ P\left(\frac{A}{R}\right) = \frac{40}{100}$$ **1**

(i) $$P(A) = P(B).P\left(\frac{A}{B}\right) + P(C).P\left(\frac{A}{C}\right) + P(R).P\left(\frac{A}{R}\right)$$

$$= \frac{10}{30} \times \frac{25}{100} + \frac{12}{30} \times \frac{35}{100} + \frac{8}{30} \times \frac{40}{100}$$

$$= \frac{990}{3000} \text{ or } \frac{33}{100}$$ **1**

(ii) **(a)** $$P\left(\frac{C}{A}\right) = \frac{P(C).P\left(\frac{A}{C}\right)}{P(B).P\left(\frac{A}{B}\right)+P(C).P\left(\frac{A}{C}\right)+P(R).P\left(\frac{A}{R}\right)}$$

$$= \frac{\frac{12}{30}\times\frac{35}{100}}{\frac{990}{3000}}$$ **1**

$$= \frac{42}{99} \text{ or } \frac{14}{33}$$ **1**